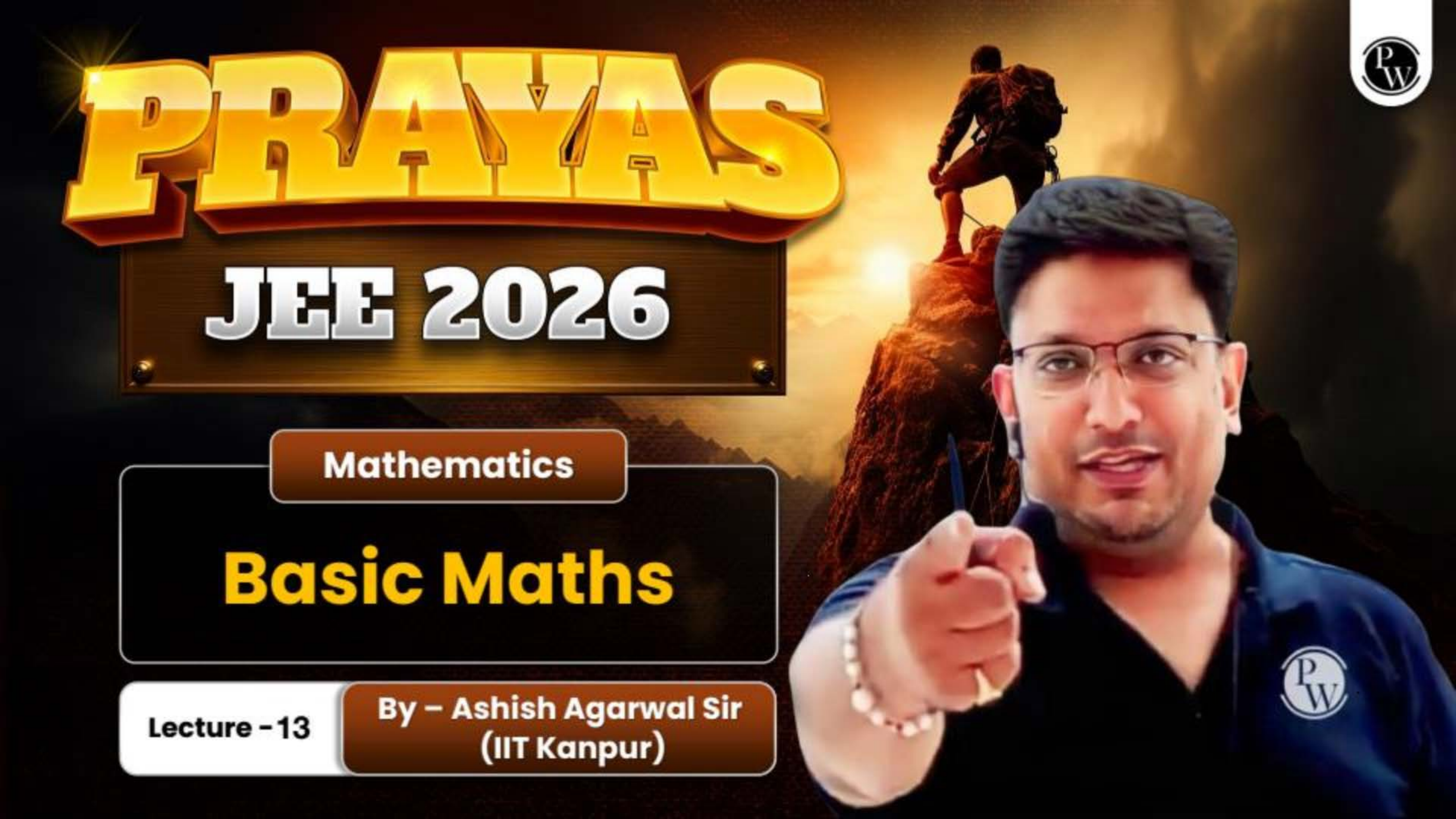

PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture - 13
By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Introduction to Modulus

B Problem Practice

Homework Discussion

QUESTION [JEE Mains 2023]

The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

$\left(\frac{x-7}{2x-3}\right)^2 > 0$

$x \neq 7$ — A

$x \neq \frac{3}{2}$

(A) 8

(B) 7

(C) 5

(D) 6

Case ① if $x+\frac{7}{2} > 1 \Rightarrow x > -5/2$

$\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$\frac{(x-7)^2}{(2x-3)^2} - 1 \geq 0$

$\frac{(x-7)^2 - (2x-3)^2}{(2x-3)^2} \geq 0$

$\frac{(3x-10)(-x-4)}{(2x-3)^2} \geq 0$

Case ⑪ if $0 < x+\frac{7}{2} < 1$

$-\frac{7}{2} < x < -5/2$

$\frac{(x-7)^2}{(2x-3)^2} \leq 1$

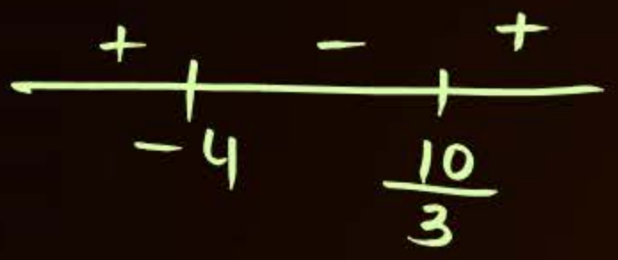
$\frac{(3x-10)(-x-4)}{(2x-3)^2} \leq 0$

$(3x-10)(x+4) \geq 0 \quad x \neq 3/2$

$x \in (-\infty, -4] \cup [10/3, \infty)$

$\cap \quad \phi$

$$\frac{(3x-10)(x+4)}{(2x-3)^2} \le 0 , \quad x \ne \frac{3}{2}$$



$$x \in \left[-4, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$$

$$\Downarrow$$

$$x \in \left(-5/2, \frac{10}{3}\right] - \{3/2\}$$

$x \in \phi$

$\cup$

$$\left(-5/2, \frac{10}{3}\right] - \{3/2\} \text{ — (B)}$$

FINAL ANS : $A \cap B = \left(-5/2, \frac{10}{3}\right] - \{3/2\}$

Integral values of x $= -2, -1, 0, 1, 2, 3 \Rightarrow$ 6 values.

QUESTION

Solve the following inequalities:

(a) $\log(x^2 - 2x - 2) \leq 0$

(b) $\log_5(x^2 - 11x + 43) < 2$

(c) $2 - \log_2(x^2 + 3x) \geq 0$

(d) $\log_{1.5}\frac{2x-8}{x-2} < 0$

(e) $\log_3\frac{1+2x}{1+x} < 1$

(f) $\log_4\frac{3x+2}{x} \leq 0.5$

(g) $\log_2\frac{x^2-4x+2}{x+1} \leq 1$

(h) $\log_2^2\left(\frac{4x-3}{4-3x}\right) > -\frac{1}{2}$

Answers:

(a) $[-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$;

(b) $(2, 9)$;

(c) $[-4, -3) \cup (0, 1]$;

(d) $(4, 6)$;

(e) $(-\infty, -2) \cup (-1/2, \infty)$;

(f) $[-2, -2/3)$;

(g) $[0, 2-\sqrt{2}) \cup (2+\sqrt{2}, 6]$;

(h) $\left(\frac{3}{4}, \frac{4}{3}\right)$

(h) $\left(\log_2\left(\frac{4x-3}{4-3x}\right)\right)^2 \geq -\frac{1}{2}$

As $\log_2\left(\frac{4x-3}{4-3x}\right)$ should be defined

$$\frac{4x-3}{4-3x} > 0$$

$$\frac{4x-3}{3x-4} < 0$$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 3/4 & & 4/3 & \end{array}$$

$x \in (3/4, 4/3)$ Ans

Home Challenge-04

JEE MAINS 10 Apr Morning SHIFT 2023

Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then, $6a + 5bc$ is equal to

$$\ln (2a)^{\ln a} = \ln (bc)^{\ln b}$$

$$\ln a \cdot (\ln 2a) = \ln b \cdot \ln bc$$

$$\ln a (\ln 2 + \ln a) = \ln b (\ln b + \ln c)$$

$$\ln 2 \cdot \ln a + (\ln a)^2 = (\ln b)^2 + \frac{\ln b \cdot \ln 2 \cdot \ln b}{\ln a}$$

$$\ln b^{\ln 2} = \ln a^{\ln c}$$

$$\ln 2 \cdot \ln b = \ln c \cdot \ln a$$

$$\ln c = \frac{\ln 2 \cdot \ln b}{\ln a}$$

$$(\ln a)^2 - (\ln b)^2 = \frac{(\ln b)^2 \cdot \ln 2}{\ln a} - \ln 2 \cdot \ln a = \ln 2 \left(\frac{(\ln b)^2}{\ln a} - \ln a\right) = \frac{\ln 2 \left((\ln b)^2 - (\ln a)^2\right)}{\ln a}$$

$$(\ln a)^2-(\ln b)^2=\ln 2\left(\frac{(\ln b)^2-(\ln a)^2}{(\ln a)}\right)$$

$$(\ln a)^2-(\ln b)^2-\ln 2\cdot\frac{(\ln b)^2-(\ln a)^2}{\ln a}=0$$

$$\left[(\ln a)^2-(\ln b)^2\right]\left(1+\frac{\ln 2}{\ln a}\right)=0$$

$$(\ln a)^2-(\ln b)^2=0 \quad \text{or} \quad \frac{\ln 2}{\ln a}+1=0$$

$$(\ln a-\ln b)(\ln a+\ln b)=0$$

$$\ln a=\ln b \text{ or } \ln a+\ln b=0$$

$$a=b \quad \text{or} \quad \ln a=-\ln b=\ln\frac{1}{b}$$

$\to$ (N.P) $\qquad a=\frac{1}{b}$

$$\ln 2=-\ln a$$

$$\ln a=\ln 1/2$$

$$a=1/2\Rightarrow \ln c=\frac{\ln 2\cdot\ln b}{\ln a}=\frac{\ln 2\cdot\ln b}{-\ln 2}=-\ln b\Rightarrow \ln b+\ln c=0$$

$$\ln bc=0\Rightarrow bc=1$$

$$\cancel{x^2}=\cancel{x}$$

$$x=1$$

Gadho/Gadhiyoo aisaa naa karo!!

Sahi

$$x^2=x$$

$$x^2-x=0\Rightarrow x(x-1)=0$$

$$x=0,1$$

If $a = \frac{1}{b}$

$$\ln c = \frac{\ln 2 \cdot \ln b}{\ln a}$$

$$\ln c = \frac{\ln 2 \cdot \cancel{\ln b}}{-\cancel{\ln b}} = -\ln 2$$

$$c = \frac{1}{2}$$

$6a + 5bc$ has ∞ many possible values.

$$(2a)^{\ln a} = (bc)^{\ln b}, \quad b^{\ln 2} = a^{\ln c} \to b^{\ln 2} = \left(\frac{1}{b}\right)^{\ln 1/2} = (b^{-1})^{-\ln 2} = b^{\ln 2}$$

$$\left(\frac{2}{b}\right)^{\ln \frac{1}{b}} = \left(\frac{b}{2}\right)^{\ln b}$$

$$\left(\frac{2}{b}\right)^{-\ln b} = (b/2)^{\ln b} \Rightarrow (b/2)^{\ln b} = (b/2)^{\ln b}$$

Hence

$$6a + 5bc = 6 \cdot \frac{1}{2} + 5 = 8 \text{ Ans}$$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION [JEE Mains 2021]

The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is Tah 01

QUESTION

Find x for $\frac{(\ln x)^2-3\ln x+3}{\ln x-1}<1.$

$\ln x = t$

$$\frac{t^2-3t+3}{t-1}-1<0$$

$$\frac{t^2-4t+4}{t-1}<0$$

$$\frac{(t-2)^2}{(t-1)}<0$$

$$\frac{1}{t-1}<0 \qquad t \neq 2$$

$$t-1<0$$

$$t<1$$

$\ln x<1$ & $x>0$, $\ln x \neq 2$

$\log_e x<1$

$x<e$

$x \neq e^2$

$x \in (0,e)$

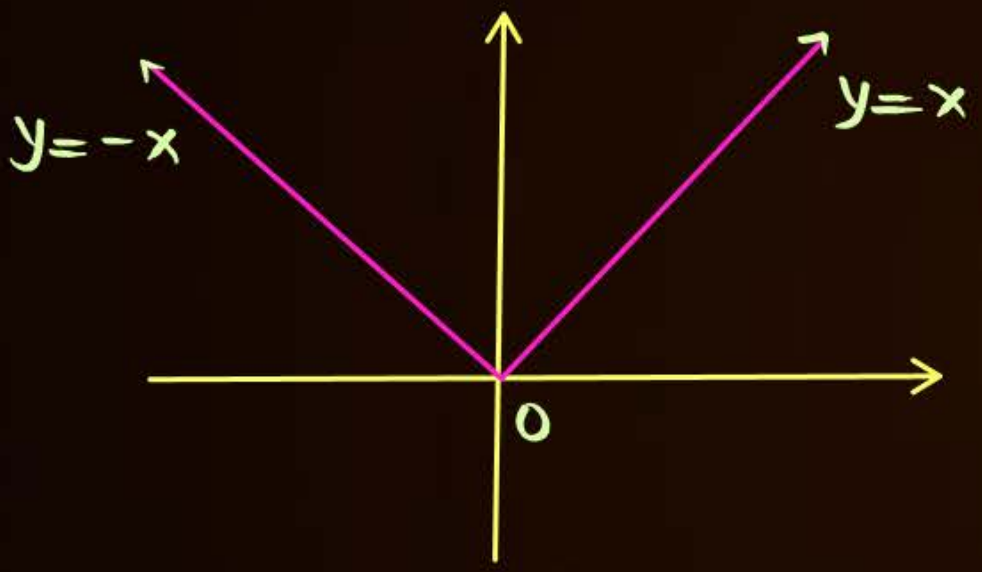
QUESTION

Solve : $\log_{\frac{1}{x}}\left(\frac{2(x-2)}{(x+1)(x-5)}\right) \geq 1$

Tah02

Modulus/Absolute value Function

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

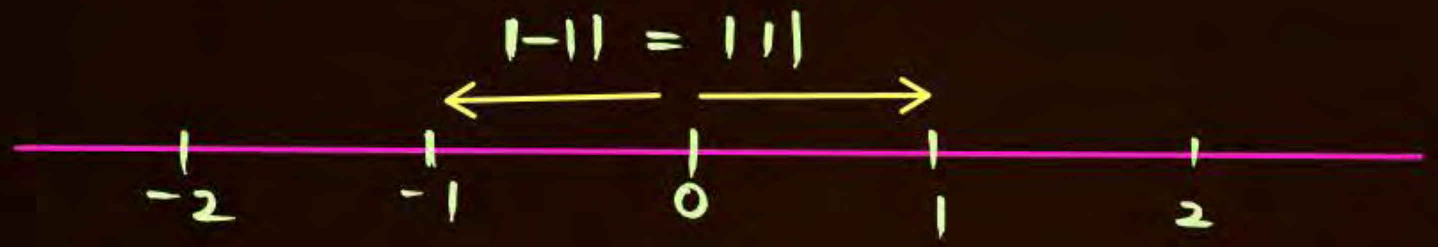


★ $|x| \geq 0$

★ $|x| = 0 \iff x = 0$

Geometrical Meaning of Modulus

$|x|$ = distance of x on number line from 0



$|x| = a \; , \; a \geq 0$

$\Downarrow$

$x = \pm a$

Ex: $|x| = 2$

$x = -2, 2$

Ex: $|2x-1| = 5$

$2x-1 = -5, 5.$

$2x = -4, 6$

$x = -2, 3$

Ex: $|2x-6| = -10$

(No soln)

Important Properties

P_1: $|-x| = |x|$ ★★★★

P_2: $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

Modulus distributes over multiplication & division

$|x_1 . x_2 \ldots x_n| = |x_1| . |x_2| \ldots |x_n|$

P_3: $|xy| = |x||y|$

Important Properties

P_4: $\sqrt{x^2} = |x|, x \in R$, Also $x^2 = |x|^2$

Yaad Rakhe (varaible/Expression)^2 OR

Ko |variable/Expression|^2

likh sakte hai

P_5: $|x| \geq 0$ also $|x| \geq x$

- $|x| > x \longrightarrow x \in R^-$
- $|x| = x \longrightarrow x \in [0, \infty)$

(i) $|f(x)| = a \quad , a \geq 0$

$f(x) = \pm a$

(ii) $|f(x)| = g(x)$

case① $f(x) \geq 0$

$\downarrow$

solve $f(x) = g(x)$

case⑪ $f(x) < 0$

$-f(x) = g(x)$

$\cap$

$\cap$

UNION

(FINAL Ans)

$|f(x)| = |g(x)|$

$\Downarrow$

$(f(x))^2 = (g(x))^2$

Apply $a^2 - b^2$

$\parallel$

$(a-b)(a+b)$

QUESTION

$$|x^2 + x - 20| = -(x^2 + x - 20)$$

$\Downarrow$

$x^2+x-20 \le 0$

$\Downarrow$

$(x+5)(x-4) \le 0$

$x \in [-5, 4]$

$|x| = -x$, $|x| = x$

$\Downarrow$ $\Downarrow$

$x \le 0$ $x \geq 0$

QUESTION

$$\left|\left(\frac{x^2-6x+8}{x^2-4x+3}\right)\right| = -\left(\frac{x^2-6x+8}{x^2-4x+3}\right)$$

Tah03

QUESTION

Solve: $|2x - 1| = 7$

$2x - 1 = -7, 7$

$2x = -6, 8$

$x = -3, 4$

QUESTION

Solve: $|3 - x| = 2$

$3 - x = -2, 2$

$-x = -5, -1$

$x = 5, 1$

QUESTION

$|x + 2| = 2(3 - x)$ then x is equal to

(A) 4/3

(B) –8

(C) 8

(D) –4/3

Case (I) If $x+2 \geq 0 \Rightarrow x \geq -2$

$x+2 = 6-2x$

$3x = 4$

$x = \frac{4}{3}$

$x = \frac{4}{3}$

Case (II) If $x+2 < 0 \Rightarrow x < -2$

$-x-2 = 6-2x$

$x = 8$

$x \in \phi$

$x = \frac{4}{3}$

QUESTION

If $|3x - 2| + x = 11$ then x is Jan 04

(A) 13/4

(B) 9/2

(C) −9/2

(D) −13/4

QUESTION

$\left|\left||x-2|-2\right|-2\right| = 2$ then sum of all values of x satisfying the equation is

(A) 8

(B) 12

(C) 0

(D) 4

$$\Big|\underbrace{\big||x-2|-2\big|}_{m}-2\Big| = 2$$

$|m-2| = 2$

$m-2 = -2, 2$

$m = 0, 4$

$m = 0$

$\big||x-2|-2\big| = 0$

$|x-2|-2 = 0$

$|x-2| = 2$

$x-2 = -2, 2$

$x = 0, 4$

$m = 4$

$\big||x-2|-2\big| = 4$

$|x-2|-2 = -4, 4$

$|x-2| = -2, 6$

$|x-2| = -2$ or $|x-2| = 6$

Not possible

$x-2 = 6, -6$

$x = 8, -4$

$x = 0, 4, 8, -4$

$Sum = 0 + 4 + 8 - 4 = 8$

QUESTION

Solve: $\underset{F_1}{|x + 1|} + \underset{F_2}{|x - 2|} = 5$

	$x<-1$	$-1<x<2$	$x>2$
F_1	$-ve$	$+ve$	$+ve$
F_2	$-ve$	$-ve$	$+ve$

Case (I) If $x \geq 2$

$x+1+x-2=5$

$2x=6$

$x=3$

$x=3$

Case (II) $-1<x<2$

$x+1-(x-2)=5$

$3=5$

$\downarrow$

Not possible

Case (III) If $x \leq -1$

$-(x+1)-(x-2)=5$

$-2x+1=5$

$x=-2$

$\cap$

$x=-2$

UNION

Ans $x=-2, 3$

QUESTION

$|x-1| + |x-2| + |x-3| = 9$ then x can be

(A) –5

(B) 9

(C) –1

(D) 5

Tan 05

T_1	–ve	+ve	+ve	+ve
T_2	–ve	–ve	+ve	+ve
T_3	–ve	–ve	–ve	+ve

1 2 3

Case (1) if $x \le 1$

$-(x-1)-(x-2)-(x-3)=9$

$-3x+6=9$

$x=-1$

$x=-1$

Case (11) if $1 < x \le 2$

Case (111)

QUESTION [JEE Mains 2021]

The number of real solution of the equation, $x^2 - |x| - 12 = 0$ is:

(A) 2

(B) 3

(C) 1

(D) 4

M (1)

case (1) If $x \geq 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = -3, 4$

$x = 4$

case (ii) If $x < 0$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, 3$

$x = -4$

$x = -4, 4$

M (2) $x^2 - |x| - 12 = 0$

$|x|^2 - |x| - 12 = 0$

$|x| = t \quad t^2 - t - 12 = 0$

$t = 4, -3$

$|x| = 4, \cancel{-3}$

$x = \pm 4$

QUESTION [JEE Mains 2025 (8 April)]

Tah 06

The sum of the squares of the roots of $|x-2|^2 + |x-2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x-3| - 5 = 0$, is

$|x-2| = t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2, 1$

$|x-2| = \cancel{-2}, 1$

(A) 24

(B) 26

(C) 36

(D) 30

Ans. C

QUESTION [JEE Mains 2025 (7 April)]

The number of real roots of the equation $x|\overset{T_1}{x-2}| + 3|\overset{T_2}{x-3}| + 1 = 0$ is: Tah 07

T_1	$-ve$		$+ve$		$+ve$
T_2	$-ve$	2	$-ve$	3	$+ve$

(A) 4

(B) 3

(C) 2

(D) 1

case 1

Ans. D

QUESTION [JEE Mains 2024 (5 April)]

The number real solutions of the equations $x|x+5|+2|x+7|-2=0$ is

Tan 08

Ans. 3

QUESTION [JEE Mains 2024 (5 April)]

Tah 09

The number of distinct real roots of the equation $\overset{T_1}{|x|}\ \overset{T_2}{|x+2|} - 5\overset{T_3}{|x+1|} - 1 = 0$ is

	$(-\infty,-2)$	$(-2,-1)$	$(-1,0)$	$(0,\infty)$
T_1	$-ve$	$-ve$	$-ve$	$+$
T_2	$-ve$	$+ve$	$+ve$	$+$
T_3	$-ve$	$-ve$	$+ve$	$+$

Case ① If $x \le -2$

$-x \cdot -(x+2) + 5(x+1) - 1 = 0$

$x^2 + 2x + 5x + 5 - 1 = 0$

$x^2 + 7x + 4 = 0$

$x = \dfrac{-7 \pm \sqrt{49-16}}{2} = \dfrac{-7 \pm \sqrt{33}}{2}$

$x = \dfrac{-7-\sqrt{33}}{2}$

$\dfrac{-7+\sqrt{33}}{2}$ ⊙ -2

$-7+\sqrt{33}$ ⊙ -4

$\sqrt{33}$ ⊙ 3

Ans. 3

QUESTION [JEE Mains 2024 (8 April)]

$-4+1$

The number of distinct real roots of the equation $\underbrace{|x+1|}_{T_1}\,\underbrace{|x+3|}_{T_2} - 4\underbrace{|x+2|}_{T_3} + 5 = 0$ is

	$-\infty$ to -3	-3 to -2	-2 to -1	-1 to ∞
T_1	$-ve$	$-ve$	$-ve$	$+ve$
T_2	$-ve$	$+ve$	$+ve$	$+ve$
T_3	$-ve$	$-ve$	$+ve$	$+ve$

Case (i) If $x \le -3$

Case (ii) If $-3 < x \le -2$

Case (iii) If $-2 < x < -1$

Case (iv) If $x \ge -1$

Thumbs up — Yes understood — 96%

Thumbs down — Abhi bhi nahi aaya — 4%

No: of Number ke Tukde
||
No: of cases

Chotaa NO: — Baada NO:

Ans. 2

QUESTION [JEE Mains 2023]

The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is:

(A) $11 + \sqrt{3}$

(B) $9 + \sqrt{3}$

(C) $9 - \sqrt{3}$

(D) $11 - \sqrt{3}$

Case (I) If $x^2 - 8x + 15 > 0$

$(x-3)(x-5) > 0$

$x \in (-\infty, 3) \cup (5, \infty)$

$x^2 - 8x + 15 - 2x + 7 = 0$

$x^2 - 10x + 22 = 0$

$x = \frac{10 \pm \sqrt{12}}{2}$

$x = 5 \pm \sqrt{3}$

$\cap$ $x = 5 + \sqrt{3}$

Case (II) If $x^2 - 8x + 15 \le 0$

$(x-5)(x-3) \le 0$

$x \in [3, 5]$

$-(x^2 - 8x + 15) - 2x + 7 = 0$

$-x^2 + 6x - 8 = 0$

$x^2 - 6x + 8 = 0$

$x = 2, 4$

$\cap$ $x = 4$

UNION

$x = 4, 5 + \sqrt{3}$

Sum $= 9 + \sqrt{3}$.

QUESTION [JEE Mains 2024 (30 Jan)]

The number of real solutions of the equation $x(x^2 + 3|x| + 5|x - 1| + 6|x - 2|) = 0$ is ______

$x = 0$ or $\underbrace{x^2}_{\geq 0} + \underbrace{3|x|}_{\geq 0} + \underbrace{5|x-1|}_{\geq 0} + \underbrace{6|x-2|}_{\geq 0} = 0$

$x = 0$

$x^2 = 0$ & $x = 0$ & $x - 1 = 0$ & $x = -2$ $\Rightarrow$ $x = 0$ & $x = 0$ & $x = 1$ & $x = 2$ $\cap$ $\rightarrow x \in \phi$

only one soln

No soln

Ans. 1

QUESTION [JEE Mains 2018]

Let $S = \{x \in R: x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

(A) ~~contains exactly two elements~~

(B) contains exactly four elements

(C) is an empty set

(D) contains exactly one element

$\sqrt{x} = t$

$2|t-3| + t(t-6) + 6 = 0$

Case(1) $t \geq 3$

$2t - 6 + t^2 - 6t + 6 = 0$

$t^2 - 4t = 0$

$t = 0, 4$

n

$t = 4$

Case(11) if $t < 3$

$-2t + 6 + t^2 - 6t + 6 = 0$

$t^2 - 8t + 12 = 0$

$t = 2, 6$

n

$t = 2$

$t = 4, 2$

$\sqrt{x} = 4, 2$

$x = 16, 4$

$S = \{16, 4\}$

QUESTION [JEE Mains 2021]

Jan 09

The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's BPP

Lo Karo Duvaadaar Practice!!

1. $\log_5(x^2 - 3x + 3) > 0$

2. $\log_7[\log_5(x^2 - 7x + 15)] > 0$

3. $\log_{\left(\frac{1}{2}\right)}[\log_5(x^2 - 7x + 17)] > 0$

4. $\log_{\left(\frac{1}{2}\right)}(\log_5(\log_2(x^2 - 6x + 40))) > 0$

5. $\log_3[\log_5 \log_2(x^2 - 9x + 50)] > 0$

6. $\log_6\left(\frac{x-2}{6-x}\right) > 0$

7. $\log_{0.5}(x^2 - 5x + 6) > -1$

8. $\log_8(x^2 - 4x + 3) < 1$

9. $\log_{\left(\frac{1}{4}\right)}\left(\frac{35-x^2}{x}\right) \geq -\frac{1}{2}$

Answers

1. $(-\infty, 1) \cup (2, \infty)$
2. $(-\infty, 2) \cup (5, \infty)$
3. $(3, 4)$
4. $(2, 4)$
5. $(-\infty, 3) \cup (6, \infty)$
6. $(4, 6)$
7. $(1, 4)$
8. $(-1, 5)$
9. $(-1, 0) \cup (5, \infty)$

Home Challenge-05

If the value of x which satisfies the equation $2\log_3\sqrt{3^{1-x}+2} = 1+\log_3(4\cdot 3^x-1)$ is given by, $1-\log_3 k$, then find the value of k. [Ans. 4]

Homework From Module

Prarambh (Topicwise) : Q1 to Q17

Prabal (JEE Main Level) : Q1 to Q7

Solution to Previous TAH

QUESTION [JEE Mains 2023]

The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

(A) 8

(B) 7

(C) 5

(D) 6

Q-3 (TAH-1): The number of integral solutions x of $\log_{(x+\frac{7}{2})}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is:

TAH-1, By Reed From WB

(a) 8 (b) 7 (c) 5 (d) 6.

Sol^n

$\left(\frac{x-7}{2x-3}\right)^2 > 0$ for log to be defined

$\Rightarrow \frac{x-7}{2x-3} \neq 0 \Rightarrow x \neq 7,\ x \neq \frac{3}{2}$ — (A)

Case-1: If $x+\frac{7}{2} > 1$

$\Rightarrow x > 1-\frac{7}{2} \Rightarrow x > -\frac{5}{2}$ — (P.C)

$\therefore \log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

$\Rightarrow \left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$\Rightarrow \frac{(x-7)^2}{(2x-3)^2} - 1 \geq 0$

$\Rightarrow \frac{(x-7)^2-(2x-3)^2}{(2x-3)^2} \geq 0$

$\Rightarrow \frac{(x-7+2x-3)(x-7-2x+3)}{(2x-3)^2} \geq 0$

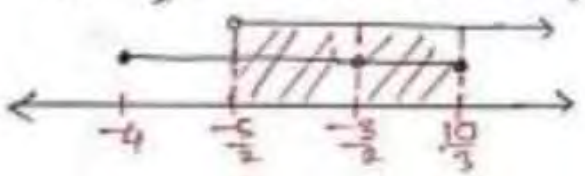
$\Rightarrow \frac{(3x-10)(-x-4)}{(2x-3)^2} \geq 0$

$\Rightarrow (3x-10)(x+4) \leq 0;\ x \neq \frac{3}{2}$

$\therefore x \in \left[-4, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$

Taking '$\cap$' with P.C.

$x > -\frac{5}{2} \cap x \in \left[-4, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$



$\therefore x \in \left(-\frac{5}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$ — (1)

TAH 1(part-1) by Reed from WB

Case-2: If $0 < x+\frac{7}{2} < 1$.

or, $-\frac{7}{2} < x < -\frac{5}{2}$ → (P.C)

now $\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

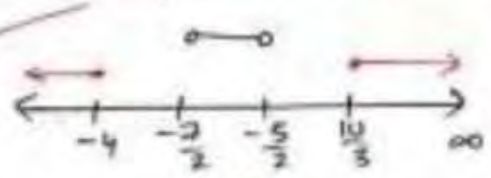
$\Rightarrow \left(\frac{x-7}{2x-3}\right)^2 \leq 1$

$\Rightarrow \frac{(3x-10)(x+4)}{(2x-3)^2} \geq 0$

$\Rightarrow (3x-10)(x+4) \geq 0;\ x \neq \frac{3}{2}$

$\therefore x \in (-\infty, -4] \cup \left[\frac{10}{3}, \infty\right)$

'$\cap$' with P.C.



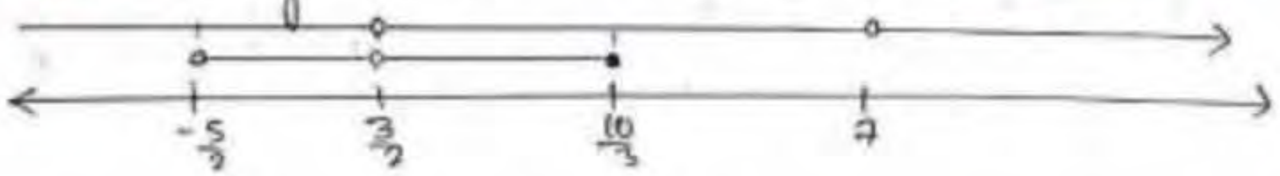
$x \in \phi$ — (ii)

TAH 1(part 2) by Reed from WB

$\therefore$ (case-i) $\cup$ (case-ii) $\Rightarrow$ Case-(1)

$x \in \left(-\frac{5}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$ — (B)

now Taking intersection of (A) & (B) (i.e. $A \cap B$)



$\therefore x \in \left(-\frac{5}{2}, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$ → final condition of 'x'.

$\therefore$ Integral values in this region

$= \underbrace{-2, -1, 0, 1, 2, 3}_{\text{6 integral values}}$.

$\therefore$ Ans $\Rightarrow$ (d) 6.

QUESTION

$$\log_{x+3}(x^2 - x) < 1$$

Tah 2

$\log_{x+3}(x^2-x) < 1$

Also $x+3>0$

$x>-3$

No Need

Also $x^2-x>0$

$x(x-1)>0$

$x \in (-\infty,0) \cup (1,\infty)$ — ①

case I : if $x+3>1 \Rightarrow x>-2$

$x^2-x < x+3$

$x^2-2x-3<0$

$(x-3)(x+1)<0$

$x \in (-1,3)$ $\cap$

$x \in (-1,3)$ — ②

Case II: $0<x+3<1$

$-3<x<-2$

$x^2-x > x+3$

$x^2-2x-3>0$

$(x-3)(x+1)>0$

$x \in (-\infty,-1) \cup (3,\infty)$ $\cap$ $x \in (-3,-2)$ — ③

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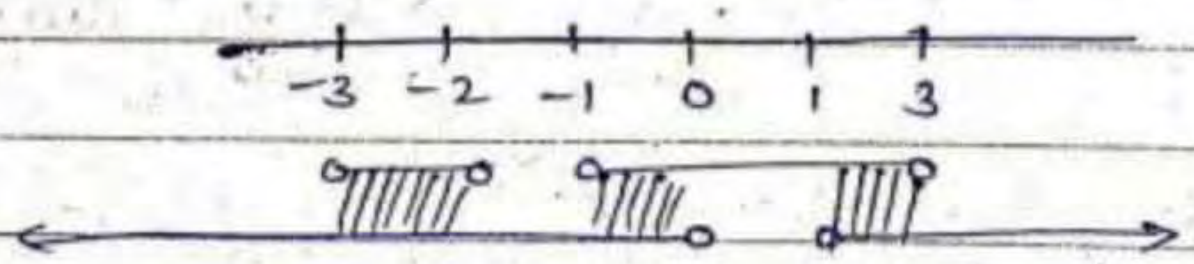
② $\cup$ ③

$x \in (-3,-2) \cup (-1,3) \cap$ ①



$x \in (-3,-2) \cup (-1,0) \cup (1,3)$ Ans

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TAH 2) $\log_{x+3}(x^2-x) < 1$

Case 1: If $x+3>1$

$x>-2$

$x \in (-2, \infty)$ — (A)

$x^2 - x < x+3$

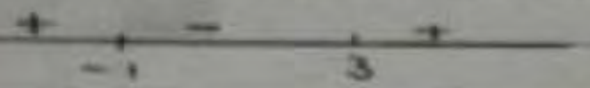
$x^2 - x - x - 3 < 0$

$x^2 - 2x - 3 < 0$

$x^2 - 3x + x - 3 < 0$

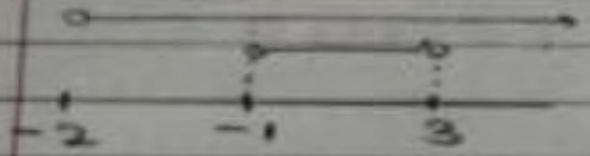
$x(x-3) + 1(x-3) < 0$

$(x+1)(x-3) < 0$



$x \in (-1, 3)$ — (B)

(A) $\cap$ (B)



$x \in (-1, 3)$ — (E)

Case 2: If $0 < x+3 < 1$

$-3 < x < -2$

$x \in (-3, -2)$ — (C)

$x^2 - x > x+3$

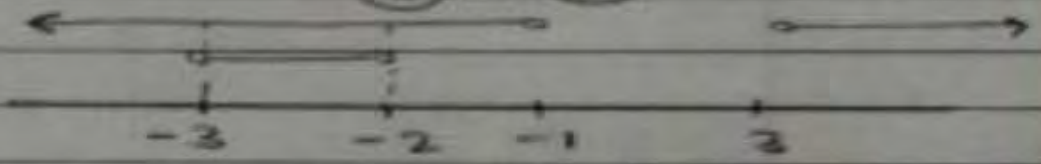
$x^2 - x - x - 3 > 0$

$x^2 - 2x - 3 > 0$

$(x+1)(x-3) > 0$

$x \in (-\infty, -1) \cup (3, \infty)$ — (D)

(C) $\cap$ (D)



$x \in (-3, -2)$ — (F)

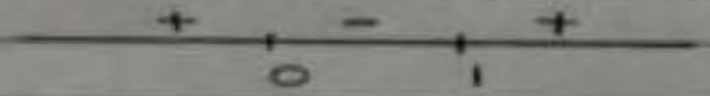
(E) $\cup$ (F)

$x \in (-3, -2) \cup (-1, 3)$ — (G)

Now,

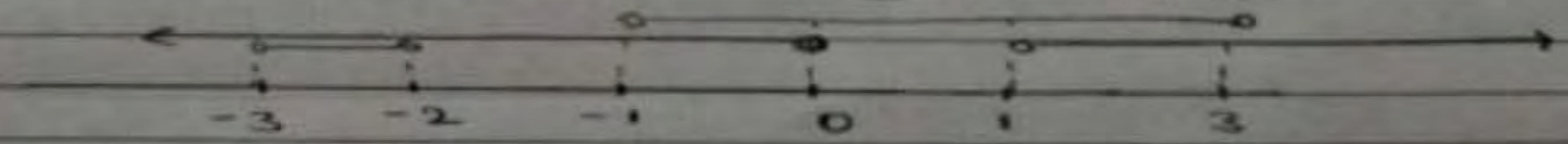
$x^2 - x > 0$

$x(x-1) > 0$



$x \in (-\infty, 0) \cup (1, \infty)$ — (H)

(G) $\cap$ (H)



$x \in (-3, -2) \cup (-1, 0) \cup (1, 3)$ **Ans.**

TAH 2
by Reed
From WB

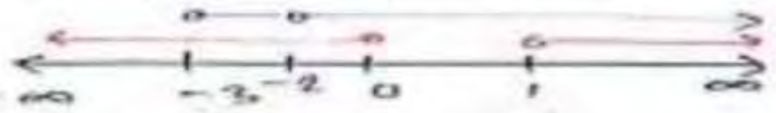
TAH-2: $\log_{x+3}(x^2-x) < 1$.

Solve: $\log_{x+3}(x^2-x) < 1$

for log to be defined,

$x+3>0 \Rightarrow x>-3$ | $x+3 \neq 1 \Rightarrow x \neq -2$ | $x^2-x>0 \Rightarrow x(x-1)>0 \Rightarrow x \in (-\infty,0) \cup (1,\infty)$

'$\cap$'



$\therefore x \in (-3,0) \cup (1,\infty) - \{-2\}$ — (A)

Case-1: If $x+3>1 \Rightarrow (x>-2)$ — (P.C)

So, $\log_{(x+3)}(x^2-x)<1$

$\Rightarrow (x^2-x) < x+3$

$\Rightarrow x^2-2x-3<0$

$\Rightarrow (x-3)(x+1)<0$

$\therefore x \in (-1,3)$ — (i) [Satisfies P.C]

Case-2: If $x+3 \in (0,1)$.

$\Rightarrow 0 < x+3 < 1$

$\Rightarrow -3 < x < -2$ $\quad \therefore x \in (-3,-2) \longrightarrow$ (P.C)

So, $\log_{x+3}(x^2-x) < 1$

$\Rightarrow (x^2-x) > (x+3)$

$\Rightarrow x^2-2x-3>0$

$\Rightarrow (x-3)(x+1)>0$

$\Rightarrow \therefore x \in (-\infty,-1) \cup (3,\infty)$

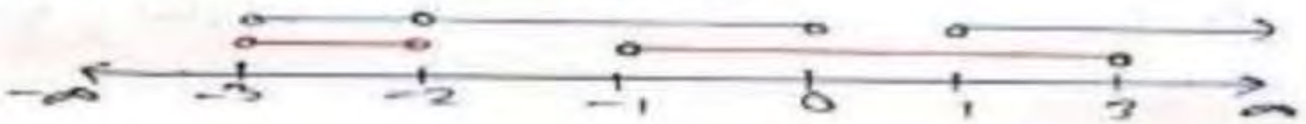
'$\cap$'

$\therefore x \in (-3,-2)$ — (ii)

Taking (i) $\cup$ (ii) $\Rightarrow$ $x \in (-3,-2) \cup (-1,3)$ — (B)

Taking intersection of (A) & (B);

$A \cap B \Rightarrow$



$\therefore x \in (-3,-2) \cup (-1,0) \cup (1,3) \longrightarrow$ Final Ans.

Solution to Previous BPPs

QUESTION

Solve the following inequalities:

(a) $\log(x^2 - 2x - 2) \leq 0$

(b) $\log_5(x^2 - 11x + 43) < 2$

(c) $2 - \log_2(x^2 + 3x) \geq 0$

(d) $\log_{1.5} \frac{2x-8}{x-2} < 0$

(e) $\log_3 \frac{1+2x}{1+x} < 1$

(f) $\log_4 \frac{3x+2}{x} \leq 0.5$

(g) $\log_2 \frac{x^2-4x+2}{x+1} \leq 1$

(h) $\log_2^2 \left(\frac{4x-3}{4-3x}\right) > -\frac{1}{2}$

Answers:

(a) $[-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$;

(b) $(2,9)$;

(c) $[-4,-3) \cup (0,1]$;

(d) $(4,6)$;

(e) $(-\infty,-2) \cup (-1/2,\infty)$;

(f) $[-2,-2/3)$;

(g) $[0, 2-\sqrt{2}) \cup (2+\sqrt{2}, 6]$;

(h) $\left(\frac{3}{4}, \frac{4}{3}\right)$

· **BPPs:-** Solve the following ineq.s.

Sol$^n \Rightarrow$ (a) $\log(x^2-2x-2) \le 0$

$\Rightarrow x^2-2x-2 \le 1$

$\Rightarrow x^2-2x-3 \le 1$

$\Rightarrow (x-3)(x+1) \le 0$

$\Rightarrow x \in [-1, 3]$ — (i)

& $x^2-2x-2 \ge 0$

$D = 4+8 = 12$

$\therefore x = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

$(x-(1+\sqrt{3}))\cdot(x-(1-\sqrt{3})) \ge 0$

$\therefore x \in (-\infty, 1-\sqrt{3}] \cup (1+\sqrt{3}, \infty)$ — (ii)

$-\infty \quad + \quad 1-\sqrt{3} \quad - \quad 1+\sqrt{3} \quad + \quad \infty$

(i) $\cap$ (ii) $\Rightarrow$

$-1 \quad 1-\sqrt{3} \quad 1+\sqrt{3} \quad 3$

$\therefore x \in [-1, 1-\sqrt{3}, 1+\sqrt{3}, 3]$

(Ans)

(b) $\log_5(x^2-11x+43) < 2$.

$\Rightarrow x^2-11x+43 < 25$

$\Rightarrow x^2-11x+18 < 0$

$\Rightarrow (x-9)(x-2) < 0$

$\Rightarrow x \in (2, 9)$

& $x^2-11x+43 > 0$

$D = 121-172 < 0$

always +ve $\quad a = 1 > 0$

$\boxed{x \in R}$

$\cap \Rightarrow \boxed{x \in 2, 9}$

→ (c) $2 - \log_2(x^2+3x) \geq 0$

$\Rightarrow \log_2(x^2+3x) \leq 2$

$\Rightarrow x^2+3x \leq 4$

$\Rightarrow x^2+3x-4 \leq 0$

$\Rightarrow (x+4)(x-1) \leq 0$

$\therefore x \in [-4, 1]$ — (i)

$x^2+3x > 0$

$\Rightarrow x(x+3) > 0$

$\Rightarrow x \in (-\infty, -3) \cup (0, \infty)$

'$\cap$'

$x \in [-4, -3) \cup (0, 1]$

Ans.

→ (d) $\log_{1.5} \frac{2x-8}{x-2} < 0$

$\Rightarrow \frac{2x-8}{x-2} - 1 < 0$

$\Rightarrow \frac{x-6}{x-2} < 0$

$\Downarrow$

$x \in (2, 6)$

&

$\frac{2x-8}{x-2} > 0$

$\Rightarrow \frac{x-4}{x-2} > 0 \Rightarrow x \in (-\infty, 2) \cup (4, \infty)$

'$\cap$'

$x \in (4, 6)$ Ans

→ (e) $\log_3 \dfrac{1+2x}{1+x} < 1$

$\Rightarrow \dfrac{1+2x}{1+x} < 3$ & $\dfrac{1+2x}{1+x} > 0$

$\Rightarrow \dfrac{1+2x-3-3x}{1+x} < 0$ | $\Downarrow$

$\Rightarrow \dfrac{x+2}{1+x} > 0$ | $x \in (-\infty, -1) \cup (-\frac{1}{2}, \infty)$ — (ii)

$\Downarrow$

$x \in (-\infty, -2) \cup (-1, \infty)$ — (i)

"∩"

$\therefore x \in (-\infty, -2) \cup (-\frac{1}{2}, \infty)$

Ans.

→ (f) $\log_4 \dfrac{3x+2}{x} \leq 0.5$

$\Rightarrow \dfrac{3x+2}{x} \leq \sqrt{4} = 2$ & $\dfrac{3x+2}{x} > 0$

$\Rightarrow \dfrac{3x+2-2x}{x} \leq 0$ | $x \in (-\infty, -\frac{2}{3}) \cup (0, \infty)$

$\Rightarrow \dfrac{x+2}{x} \leq 0$

$\therefore x \in [-2, 0)$

"∩"

$x \in [-2, -\frac{2}{3})$ Ans.

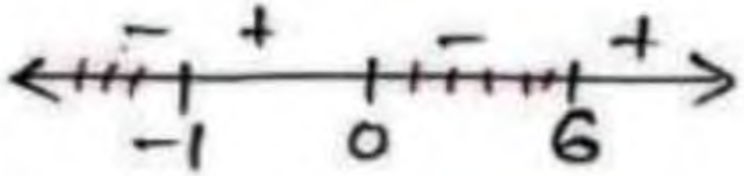
→ Q) $\cancel{\log_4 \frac{3x+2}{x}}$ $\log_2\left(\frac{x^2-4x+2}{x+1}\right) \leq 1$

$$\Rightarrow \frac{x^2-4x+2}{x+1} \leq 2$$

$$\Rightarrow \frac{x^2-4x+2-2x-2}{x+1} \leq 0$$

$$\Rightarrow \frac{x^2-6x}{x+1} \leq 0$$

$$\Rightarrow \frac{x(x-6)}{x+1} \leq 0$$



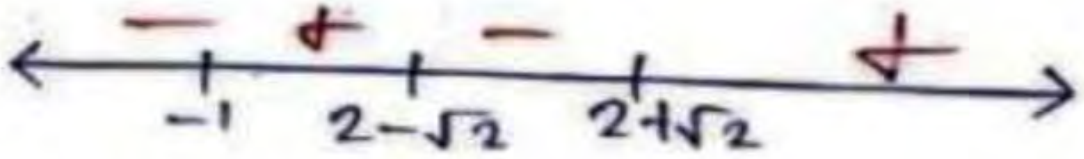
$x \in (-\infty, -1) \cup [0, 6]$

& $\frac{x^2-4x+2}{x+1} > 0$

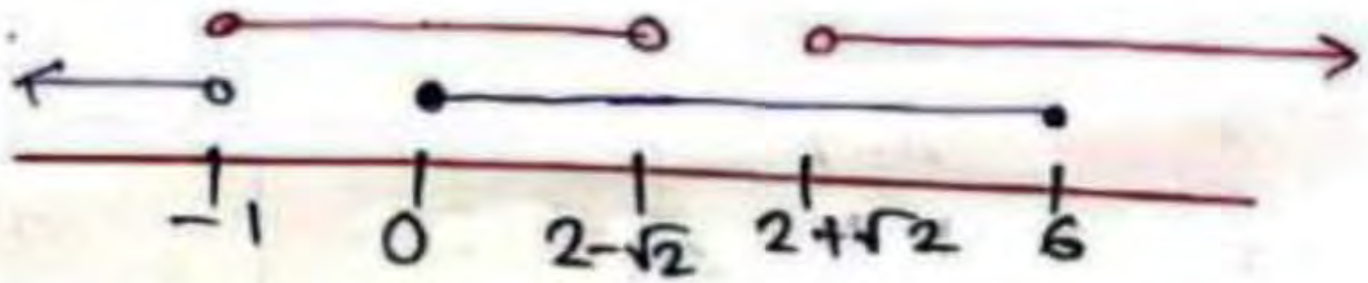
$D = 16 - 8 = 8$

$\therefore x = 2 \pm \sqrt{2}$

$$\Rightarrow \frac{(x-(2+\sqrt{2}))(x-(2-\sqrt{2}))}{x+1} > 0$$



$\therefore x \in (-1, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$



Ans ⇒ $\therefore x \in [0, 2-\sqrt{2}) \cup (2+\sqrt{2}, 6]$

Solution to Previous KTKs

QUESTION [MHT CET 2023 (9 May)]

(KTK 01)

If $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{36}$ and $x = 2^k$ then k is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{8}$

Ans. C

KTK-1. Q. If $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{36}$ and $x = 2^k$ then :

$$\Rightarrow \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{3}\log_2 x + \frac{1}{4}\log_2 = \frac{25}{36}$$

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let, $\log_2 x = t$

$$\Rightarrow t + \frac{t}{2} + \frac{t}{3} + \frac{t}{4} = \frac{25}{36} \Rightarrow \frac{12t + 6t + 4t + 3t}{12} = \frac{25}{36}.$$

$$\Rightarrow \frac{\cancel{25}t}{12} = \frac{\cancel{25}}{36} \Rightarrow t = \frac{\cancel{12}}{\cancel{36}}\frac{1}{3}$$

$$\boxed{t = \frac{1}{3}}$$

$\log_2 x = \frac{1}{3}$

$$\Rightarrow x = (2)^{1/3} \Rightarrow x = 2^k = 2^{1/3}$$

* $k = 1/3$ Ans.

QUESTION [BITSAT 2021]

(KTK 02)

If $\log_7 5 = a$, $\log_5 3 = b$ and $\log_3 2 = c$, then the logarithm of the number 70 to the base 225 is

A $\dfrac{1 - a + abc}{2a(1 + b)}$

B $\dfrac{1 - a - abc}{2a(1 + b)}$

C $\dfrac{1 + a - abc}{2a(1 + b)}$

D $\dfrac{1 + a + abc}{2a(1 + b)}$

Ans. D

Q. If $\log_7 5 = a$, $\log_5 3 = b$ and $\log_3 2 = c$, then the logarithm of the number 70 to the base 225 is :

$\log_7 5 \cdot \log_5 3 \cdot \log_3 2 = \underbrace{\log_7 2}_{(abc)}$

$\log_5 3 \cdot \log_3 2 = \underbrace{\log_5 2}_{(bc)}$

$$\Rightarrow \log_{225} 70 \Rightarrow \frac{\log_2 70}{\log_2 225}$$

$$\Rightarrow \frac{\log_2 (2\times5\times7)}{\log_2 (3\times3\times5\times5)}$$

$$\Rightarrow \frac{\log_2 2 + \log_2 5 + \log_2 7}{\log_2 3^2 + \log_2 5^2}$$

$$\Rightarrow \frac{1 + \frac{1}{\log_5 2} + \frac{1}{\log_7 2}}{2\left(\frac{1}{\log_3 2} + \frac{1}{\log_5 2}\right)}$$

$$\Rightarrow \frac{1 + \frac{1}{bc} + \frac{1}{abc}}{2\left(\frac{1}{c} + \frac{1}{bc}\right)}$$

$$\Rightarrow \frac{\frac{(bc)(abc) + (abc) + (bc)}{(bc)(abc)}}{\frac{2bc + 2c}{(c)(bc)}}$$

$$\Rightarrow \frac{\cancel{(bc)}\left[(abc) + a + 1\right]}{\cancel{(bc)}\cancel{(abc)}} \times \frac{\cancel{(c)}\cancel{(bc)}}{2\cancel{c}(1+b)}$$

$$\Rightarrow \frac{1 + a + abc}{2a(1+b)}$$ Ans.

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KTK-2.) If $\log_7 5 = a$, $\log_5 3 = b$ and $\log_3 2 = c$, then the logarithm of the number 70 to the base 225 is :

$$\log_7 5 \times \log_5 3 \times \log_3 2 = \log_7 2 = abc$$

$$\therefore \log_{225} 70 = \frac{\log_7 70}{\log_7 225} = \frac{\log_7 7 + \log_7 10}{\log_7 25 + \log_7 9}$$

$$\boxed{\log_7 5 \times \log_5 3 = \log_7 3 = ab}$$

$$= \frac{1 + \log_7 2 + \log_7 5}{2\log_7 5 + 2\log_7 3}$$

$$= \frac{1 + abc + a}{2(\log_7 5 + \log_7 3)}$$

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$$= \frac{1 + abc + a}{2(a + ab)}$$

$$\log_{225} 70 = \frac{1 + a + abc}{2a(1+b)}$$ (D) Ans.

QUESTION [BITSAT 2020]

(KTK 03)

If $\log_5 \frac{(a+b)}{3} = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4+b^4}{a^2b^2}$ is equal to

(A) 50

(B) 47

(C) 44

(D) 53

Ans. B

KTK-03. Q If $\log_5 \frac{(a+b)}{3} = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4+b^4}{a^2b^2}$ is equal to :

$\Rightarrow \quad 2\log_5 \frac{(a+b)}{3} = \log_5 (a\cdot b)$

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$\Rightarrow \quad \log_5 \left(\frac{(a+b)}{3}\right)^2 = \log_5 (a\cdot b)$

$\Rightarrow \quad a^2+b^2+2ab = 9ab$

$\Rightarrow \quad \boxed{a^2+b^2 = 7ab}$

SBS.

$\Rightarrow \quad a^4+b^4+2a^2b^2 = 49a^2b^2$

$\Rightarrow \quad a^4+b^4 = 47a^2b^2$

$\Rightarrow \quad \boxed{\frac{a^4+b^4}{a^2b^2} = 47}$ Ans.

KTK-3.) If $\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4+b^4}{a^2b^2}$ is equal to

A.) 50 B.) 47 C.) 44 D.) 53

$$\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$$

$$\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 ab}{2}$$

$$2\log_5\left(\frac{a+b}{3}\right) = \log_5 ab \Rightarrow \log_5\left(\frac{a+b}{3}\right)^2 = \log_5 ab$$

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$$\Rightarrow \left(\frac{a+b}{3}\right)^2 = ab$$

$$\Rightarrow \frac{a^2+b^2+2ab}{9} = ab$$

$$\Rightarrow a^2+b^2+2ab-9ab=0$$

$$\Rightarrow a^2+b^2 = 7ab$$

Now, $\frac{a^4+b^4}{a^2b^2} = \frac{(a^2+b^2)^2-2a^2b^2}{a^2b^2}$

$$= \frac{49a^2b^2-2a^2b^2}{a^2b^2} = \frac{47a^2b^2}{a^2b^2}$$

$$= 47 \text{ (B) Ans.}$$

QUESTION [WB JEE 2024]

(KTK 04)

If $(x^2 \log_x 27) \cdot \log_9 x = x + 4$ then the value of x is

(A) 2

(B) $-\frac{4}{3}$

(C) -2

(D) $\frac{4}{3}$

Ans. A

KTK-04. If $(x^2 \log_x 27) \cdot \log_9 x = x+4$ then the value of x is :

$$\Rightarrow x^2 \cdot \frac{\log_3 3^3}{\cancel{\log_3 x}} \cdot \frac{1}{2}\cancel{\log_3 x} = x+4$$

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$$\Rightarrow \frac{3x^2}{2} = x+4 \Rightarrow 3x^2 - 2x - 8 = 0$$

$$\Rightarrow 3x^2 - 6x + 4x - 8 = 0$$

$$\Rightarrow 3x(x-2) + 4(x-2) = 0$$

$$\Rightarrow (x-2)(3x+4) = 0$$

$$\Rightarrow x = 2, \underbrace{-\frac{4}{3}}_{\text{Reject}} \Rightarrow \boxed{x=2} \text{ Ans.}$$

KTK-4.) If $(x^2 \log_x 27) \cdot \log_9 x = x+4$ then the value of x is

A.) 2 B.) $-\frac{4}{3}$ C.) -2 D.) $\frac{4}{3}$

$\Rightarrow x^2 \log_x 27 \cdot \log_9 x = x+4$

$\Rightarrow \dfrac{x^2 \log_x 27}{\log_x 9} = x+4$

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$\Rightarrow \dfrac{x^2 \log_x 3^3}{\log_x 3^2} = x+4$

$\Rightarrow \dfrac{x^2 \times 3 \cancel{\log_x 3}}{2 \cancel{\log_x 3}} = x+4$

$\Rightarrow 3x^2 = 2x+8$

$3x^2 - 2x - 8 = 0$

$3x^2 - 6x + 4x - 8 = 0$

$3x(x-2) + 4(x-2) = 0$

$(x-2)(3x+4) = 0$

(A) $\boxed{x=2}$ $x = -\frac{4}{3}$ ✕

Ans.

QUESTION [WB JEE 2020]

(KTK 05)

If $2\log(x+1) - \log(x^2-1) = \log 2$, then $x =$

(A) only 3

(B) -1 and 3

(C) only -1

(D) 1 and 3

Ans. A

KTK-05. If $2\log(x+1) - \log(x^2-1) = \log 2$, then $x =$.

$$\Rightarrow 2\log\frac{(x+1)}{(x^2-1)} = \log 2$$

$$\Rightarrow \frac{(x+1)^{\cancel{2}}}{\cancel{(x+1)}(x-1)} = 2, \quad (x \neq -1)$$

$$\Rightarrow x+1 = 2x-2$$

$$\Rightarrow \boxed{x = 3} \text{ Ans.}$$

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KTK-5.) If $2\log(x+1) - \log(x^2-1) = \log 2$, then $x =$

$$\log(x+1)^2 - \log(x^2-1) = \log 2$$

$$\log\left(\frac{(x+1)^2}{x^2-1}\right) = \log 2$$

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$$\frac{(x+1)^2}{x^2-1} = 2$$

$$\frac{(x+1)^{\cancel{2}}}{\cancel{(x+1)}(x-1)} = 2$$

$$\frac{(x+1)}{(x-1)} = 2 \; ; \quad x \neq -1$$

$$x+1 = 2x-2$$

$$0 = 2x - x - 2 - 1$$

$$0 = x - 3 \Rightarrow \boxed{x = 3} \text{ Ans.}$$

QUESTION [WB JEE 2018]

(KTK 06)

If $x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$, then the value of x is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) 1

(D) 2

Ans. C

KTK-06. Q. If $x + \log_{10}(1+2^x) = x\log_{10}5 + \log_{10}6$, then the value of x is.

$\Rightarrow \quad x\log_{10}10 + \log_{10}(1+2^x) = \log_{10}5^x + \log_{10}6.$

$\Rightarrow \quad \log_{10}\left[10^x(1+2^x)\right] = \log_{10}(5^x \cdot 6)$

$\Rightarrow \quad \cancel{10}^x \cdot (1+2^x) = \cancel{5}^x \cdot 6$

$\Rightarrow \quad 2^x(1+2^x) = 6$ let, $2^x = t$

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$\Rightarrow \quad t(1+t) = 6$

$\Rightarrow \quad t^2+t-6 = 0 \Rightarrow (t+3)(t-2) = 0$

$\Rightarrow \quad t = -3, 2.$

$\Rightarrow \quad 2^x = -3$ | $\Rightarrow 2^x = 2$

Not possible | $\Rightarrow \boxed{x=1}$ Ans.

KTK-6.) If $x + \log_{10}(1+2^x) = x\log_{10} 5 + \log_{10} 6$, then the value of x is

A.) 1/2

B.) 1/3

C.) 1

D.) 2

Ans. $x + \log_{10}(1+2^x) = \log_{10} 5^x + \log_{10} 6$

$x + \log_{10}(1+2^x) = \log_{10} 5^x \cdot 6$

$\Rightarrow x = \log_{10} 5^x \cdot 6 - \log_{10}(1+2^x)$

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$\Rightarrow \log_{10}\left(\frac{5^x \cdot 6}{2^x+1}\right) = x$

$$\frac{5^x \cdot 6}{2^x+1} = 10^x$$

Now, by Hit & trial

at $x = 1 \Rightarrow \frac{5 \cdot 6}{2+1} = \frac{30}{3} \Rightarrow 10^1 = 10^x$ $[x=1]$ (c) Ans.

QUESTION [WB JEE 2019]

(KTK 07)

If $\log_2 6 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$, then the value of x are

(A) $\frac{1}{4}, \frac{1}{3}$

(B) $\frac{1}{4}, \frac{1}{2}$

(C) $-\frac{1}{4}, \frac{1}{2}$

(D) $\frac{1}{3}, -\frac{1}{2}$

Ans. B

KTK-07. Q. If $\log_2 6 + \frac{1}{2x} = \log_2(2^{1/x}+8)$, then the value of x are : write: $\frac{1}{2x} = \log_2(2^{1/2x})$

$$\Rightarrow \log_2(6 \cdot 2^{1/2x}) = \log_2(2^{1/x}+8)$$

$$\Rightarrow 6 \cdot 2^{1/2x} = 2^{1/x}+8 \qquad \text{let; } 2^{1/x} = t$$

$$\Rightarrow 6t^{1/2} = t+8$$

SBS

$$\Rightarrow 36t \quad t^2+16t+64$$

$$\Rightarrow t^2-20t+64=0$$

$$\Rightarrow t^2-16t-4t+64=0$$

$$\Rightarrow t(t-16)-4(t-16)=0$$

$$\Rightarrow (t-16)(t-4)=0$$

$$\Rightarrow t = 16, 4.$$

$$\Rightarrow 2^{1/x} = 16 \qquad \Rightarrow 2^{1/x} = 4$$

$$\Rightarrow \frac{1}{x} = 4 \qquad \Rightarrow \frac{1}{x} = 2$$

$$\Rightarrow \boxed{x = \frac{1}{4}} \qquad \Rightarrow \boxed{x = \frac{1}{2}}$$

Ans.

KTK-7.) If $\log_2 6 + \frac{1}{2x} = \log_2 (2^{\frac{1}{x}} + 8)$, then the value of x are

A.) $\frac{1}{4}, \frac{1}{3}$ ~~B.)~~ $\frac{1}{4}, \frac{1}{2}$ C.) $-\frac{1}{4}, \frac{1}{2}$ D.) $\frac{1}{3}, -\frac{1}{2}$

$$\log_2 6 + \frac{1}{2x} = \log_2 (2^{\frac{1}{x}} + 8)$$

$$\log_2 6 + \log_2 2^{\frac{1}{2x}} = \log_2 (2^{1/x} + 8)$$

$$\log_2 (6 \times 2^{1/2x}) = \log_2 (2^{1/x} + 8)$$

$$6 \times 2^{1/2x} = 2^{1/x} + 8$$

Let $2^{1/2x} = t$, $t^2 = 2^{1/x}$

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$$6t = t^2 + 8$$

$$t^2 - 6t + 8 = 0$$

$$t^2 - 4t - 2t + 8 = 0$$

$$t(t-4) - 2(t-4) = 0$$

$t = 2$

$2^{1/2x} = 2^1$

$\frac{1}{2x} = 1$

$2x = 1$

$\boxed{x = \frac{1}{2}}$ Ans.

$t = 4$

$2^{1/2x} = 2^2$

$\frac{1}{2x} = 2$

$\boxed{x = \frac{1}{4}}$ Ans.

(B) Ans.

QUESTION [WB JEE 2017]

(KTK 08)

If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

(A) 125

(B) 25

(C) 513

(D) 243

Ans. A

KTK-08. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$ then y equals :

$\Rightarrow \log_5 x \cdot \log_x 3x \cdot \log_{3x} y = 3.$

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$\Rightarrow \log_5 y = 3$

$\Rightarrow y = 5^3 \Rightarrow \boxed{y = 125}$ Ans.

QUESTION [COMEDK 2023]

(KTK 09)

The value of $a^{\log_b c} - c^{\log_b a}$, where $a, b, c > 0$ but $a, b, c \neq 1$, is

(A) a

(B) b

(C) c

(D) 0

Ans. D

KTK-09. The value of $a^{\log_b c} - c^{\log_b a}$, where $a, b, c > 0$ but $a, b, c \neq 1$, is :

interchange.

$\Rightarrow \quad a^{\log_b c} - a^{\log_b c}$

$\Rightarrow \quad 0$ Ans.

QUESTION [COMEDK 2022]

(KTK 10)

The value of $3^{\log_4 5} - 5^{\log_4 3}$ is

(A) 0

(B) 1

(C) 2

(D) 4

Ans. A

KTK-10. The value of $3^{\log_4 5} - 5^{\log_4 3}$ is :

$\Rightarrow 3^{\log_4 5} - 3^{\log_4 5}$ interchange.

$\Rightarrow 0$ Ans.

QUESTION [COMEDK 2021]

(KTK 11)

$8^{3\log_8 5}$ is equal to

(A) $\log_8 25$

(B) 120

(C) 125

(D) $\log_8 15$

Ans. C

KTK-11. $8^{3\log_8 5}$ is equal to :

$\Rightarrow 8^{\log_8 5^3}$

$\Rightarrow 5^3 = 125$ Ans.

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QUESTION [COMEDK 2020]

(KTK 12)

$7^{2\log_7 5}$ is equal to

(A) 5

(B) $\log_7 35$

(C) $\log_7 25$

(D) 25

Ans. D

KTK-12. $7^{2\log_7 5}$ is equal to:

$\Rightarrow 7^{\log_7 5^2}$

$\Rightarrow 5^2 = 25$ Ans.

QUESTION

(KTK 13)

Find the exhaustive solutions set of $\dfrac{(x^2-9)^{101}(x^2+6)(x^2-4)^{100}}{(x^2-5x+6)^{13}(x^2-16)^{16}} > 0.$

Ans. $(-\infty,-3) \cup (2,\infty) - \{\pm 4, 3\}$

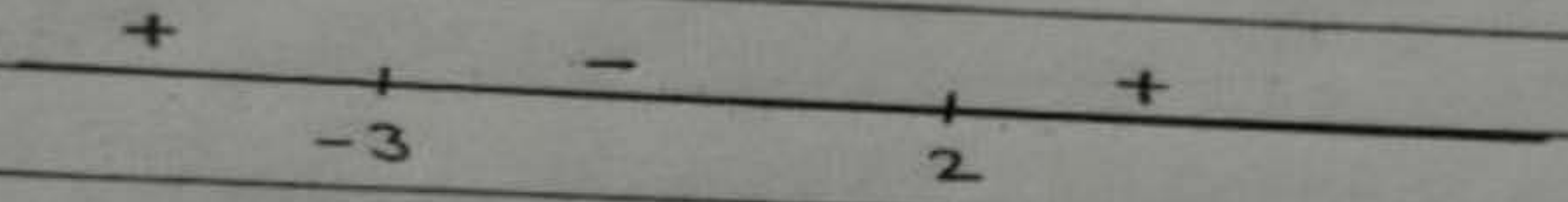
KTK-13) Find the exhaustive solutions set of $\frac{(x^2-9)^{101}(x^2+6)(x^2-4)^{100}}{(x^2-5x+6)^{13}(x^2-16)^{16}} > 0$.

$$\Rightarrow \frac{(x+3)^{101}(x-3)^{101}(x^2+6)}{(x^2-5x+6)^{13}} > 0, \quad x \neq \pm 2, \pm 4$$

$$\Rightarrow \frac{(x+3)^{101}(x-3)^{101}(x^2+6)}{(x-2)^{13}(x-3)^{13}} > 0, \quad x \neq \pm 2, \pm 4$$

$$\Rightarrow \frac{(x+3)^{101}(x-3)^{88}\overset{\text{always +ve}}{(x^2+6)}}{(x-2)^{13}} > 0, \quad x \neq \pm 2, \pm 4, 3$$

$$\Rightarrow \frac{(x+3)^{101}}{(x-2)^{13}} > 0$$



$x \in (-\infty, -3) \cup (2, \infty) - \{\pm 4, 3\}$ Ans.

QUESTION

(KTK 14)

Find the exhaustive solutions set of $\dfrac{(x-4)^{30}(x^2-9)^9(x^2-3x+2)^{17}(3x^2+10)^{10}}{(x^2-5x+6)^{52}(x^2-25)^{60}(x^2+10)^{11}} \leq 0.$

Ans. $[-3, 1] \cup (2, 3)$

KTK-14.) Find the exhaustive solutions set of

$$\frac{(x-4)^{30}(x^2-9)^9(x^2-3x+2)^{17}(3x^2+10)^{10}}{(x^2-5x+6)^{52}(x^2-25)^{60}(x^2+10)^{11}} \leq 0$$

$$\Rightarrow \frac{(x+3)^9(x-3)^9(x^2-3x+2)^{17}}{(x^2-5x+6)^{52}(x^2+10)^{11}} \leq 0 \; ; \; x \neq \pm 5 \; , \; x = 4$$

$$\Rightarrow \frac{(x+3)^9(x-3)^9(x-2)^{17}(x-1)^{17}}{(x-2)^{52}(x-3)^{52}} \leq 0 \; ; \; x \neq \pm 5$$

$$\Rightarrow \frac{(x+3)^9(x-1)^{17}}{(x-2)^{35}(x-3)^{43}} \leq 0 \; ; \; x \neq \pm 5$$

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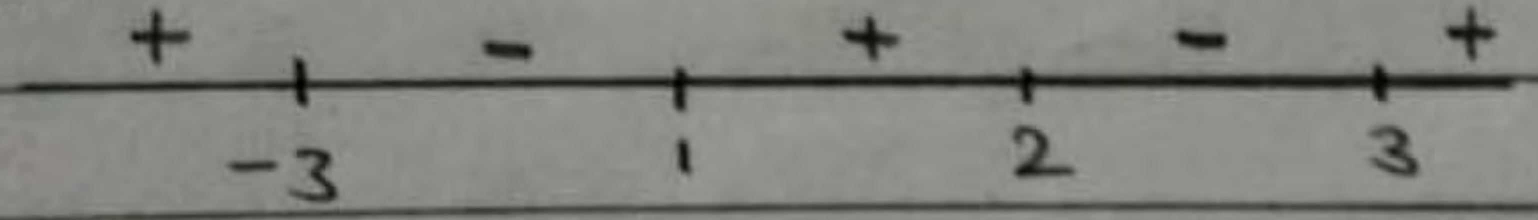

$$x \in [-3, 1] \cup (2, 3) \cup \{4\}$$

QUESTION

(KTK 15)

Solve in real numbers the equation $\sqrt{x}+\sqrt{y}+2\sqrt{z-2}+\sqrt{u}+\sqrt{v}=x+y+z+u+v$.

$$0 = x-2\cdot\sqrt{x}\cdot\frac{1}{2}+\frac{1}{4}+y-2\sqrt{y}\cdot\frac{1}{2}+\frac{1}{4}+u-2\sqrt{u}\cdot\frac{1}{2}+\frac{1}{4}+v-2\cdot\sqrt{v}\cdot\frac{1}{2}+\frac{1}{4}+\sqrt{z-2}^{2}-2\sqrt{z-2}+1+2$$

$$\cancel{-\frac{1}{4}-\frac{1}{4}-\frac{1}{4}-\frac{1}{4}-1}$$

$$0 = \left(\sqrt{x}-\tfrac{1}{2}\right)^2+\left(\sqrt{y}-\tfrac{1}{2}\right)^2+\left(\sqrt{u}-\tfrac{1}{2}\right)^2+\left(\sqrt{v}-\tfrac{1}{2}\right)^2+\left(\sqrt{z-2}-1\right)^2=0$$

$x = 1/4 = y = u = v$

$z = 3$

Ans. $x=y=u=v=1/4$, $z=3$

KTK-15: Solve in real no.s, the eqn

$$\sqrt{x}+\sqrt{y}+2\sqrt{z-2}+\sqrt{u}+\sqrt{v}=x+y+z+u+v.$$

KTK 15
by Reed
from WB

Soln

$$x+y+z+u+v=\sqrt{x}+\sqrt{y}+2\sqrt{z-2}+\sqrt{u}+\sqrt{v}$$

$$\Rightarrow x-\sqrt{x}+y-\sqrt{y}+z-2\sqrt{z-2}+u-\sqrt{u}+v-\sqrt{v}=0$$

$$\Rightarrow x-2\cdot\sqrt{x}\cdot\tfrac{1}{2}+(\tfrac{1}{2})^2+y-2\sqrt{y}\cdot\tfrac{1}{2}+(\tfrac{1}{2})^2+(z-2)-2\sqrt{z-2}\cdot 1+1^2$$
$$+u-2\sqrt{u}\cdot\tfrac{1}{2}+(\tfrac{1}{2})^2+v-2\sqrt{v}\cdot\tfrac{1}{2}+(\tfrac{1}{2})^2$$
$$=(\tfrac{1}{2})^2+(\tfrac{1}{2})^2+1^2-2+(\tfrac{1}{2})^2+(\tfrac{1}{2})^2$$

$$\Rightarrow(\sqrt{x}-\tfrac{1}{2})^2+(\sqrt{y}-\tfrac{1}{2})^2+(\sqrt{z-2}-1)^2+(\sqrt{u}-\tfrac{1}{2})^2+(\sqrt{v}-\tfrac{1}{2})^2$$
$$=\tfrac{1}{4}+\tfrac{1}{4}+1+\tfrac{1}{4}+\tfrac{1}{4}-2=2-2=0.$$

$\Rightarrow \sqrt{x}=\frac{1}{2}$, or, $x=\frac{1}{4}$ | $\sqrt{y}=\frac{1}{2}$, $y=\frac{1}{4}$ | $\sqrt{z-2}=1$, $z-2=1$, $z=3$ | $\sqrt{u}=\frac{1}{2}$, $u=\frac{1}{4}$ | $\sqrt{v}=\frac{1}{2}$, $v=\frac{1}{4}$

$$\therefore \boxed{x=y=u=v=\tfrac{1}{4}}\ \&\ \boxed{z=3}$$

QUESTION

(KTK 16)

Find all pair of positive integer (m, h) that satisfy $mn + 3m - 8n = 59$.

Ans. 3

KTK-16.) Find all pair of positive integer (m, n) that satisfy

$$mn + 3m - 8n = 59$$

$$\begin{array}{r} 59 \\ 24 \\ \hline 35 \end{array}$$

Ans. $m(n+3) - 8n = 24 + 35$

$m(n+3) - 8n - 24 = 35$ $\qquad 1 \times 35,\ 5 \times 7,$

$m(n+3) - 8(n+3) = 35$ $\qquad 7 \times 5,\ 35 \times 1$

$(m-8)(n+3) = 35$

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$m-8$	$n+3$	(m, n)
1	35	$(9, 32)$
5	7	$(13, 4)$
7	5	$(15, 2)$
35	1	$(43, -2)$ ×

$= 3$ pairs

KTK-17.) The least ...

KTK-16: Find all pair of +ve integer (m,n) that $mn+3m-8n=59$.

KTK 16
by Reed
from WB

Soln: $mn+3m-8n=59$

or, $m(n+3)-8n-24=59-24$

or, $m(n+3)-8(n+3)=35$

or $(m-8)(n+3)=35$.

now, $35=5\times7$ | $35=(-5)\times(-7)$

$35=1\times35$ | $\underbrace{35=(-1)\times(-35)}$

no need since $m,n\in I^+$

$m-8=$	5	7	1	35
$n+3=$	7	5	35	1
$\Rightarrow m=$	13	15	9	43
& $n=$	4	2	32	-2

✗ ($\because m,n\in I^+$)

$\therefore$ all pair of positive integers $(m,n)\equiv(13,4),(15,2),(9,32)$ (Ans.)

$\therefore$ no. of pairs of (m,n) is $=3$.

QUESTION

(KTK 17)

The least value of the expression $(x + y)(y + z)$ where given that $x, y, z > 0$ and $xyz(x + y + z) = 1$

Ans. 2

KTK-17.) The least value of the expression $(x+y)(y+z)$ where given that $x, y, z > 0$ and $xyz(x+y+z) = 1$

Ans. $x+y+z = \frac{1}{xyz}$

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$(x+y)(y+z)$

$xy + xz + y^2 + yz$

$y(x+y+z) + xz$

$\Rightarrow y\left(\frac{1}{xyz}\right) + xz$

$\Rightarrow \frac{1}{xz} + xz$

as $\frac{1}{xz} + xz \geq 2$

Least value $= \underline{2}$ Ans.

KTK-17: The least value of the expression $(x+y)(y+z)$ where given that $x, y, z > 0$ and $xyz(x+y+z) = 1$.

Sol^n:

$$E = (x+y)(y+z)$$

$$\Rightarrow E = xy + xz + y^2 + yz$$

$$\Rightarrow E = y(x+y+z) + xz$$

$$\Rightarrow E = \frac{\cancel{y}}{x\cancel{y}z} + xz \qquad \because y \neq 0$$

$$\Rightarrow E = \underbrace{\frac{1}{xz} + xz}_{\geq 2}. \qquad \because x, y, z > 0 \quad \therefore xz > 0$$

$$\therefore E_{min}\Big|_{at\ x=z=1} = 2$$

$$\therefore \boxed{E_{min} = 2} \text{ Ans.}$$

$xyz(x+y+z) = 1$

$\Rightarrow (x+y+z) = \dfrac{1}{xyz}$

KTK 17
by Reed
from WB

* Read class Theory
* Retry class Questions In rough Copy
* TAH
* BPP
* KTK
* DPP/Module.

THANK YOU